Paper Reference(s)

6678

Edexcel GCE

Mechanics M2

Advanced

Friday 11 June 2010 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

N35391A

1. A particle P moves on the x-axis. The acceleration of P at time t seconds, $t \geq 0$, is $(3t + 5)$ m s^{-2} in the positive x-direction. When $t = 0$, the velocity of P is 2 m s^{-1} in the positive x-direction. When $t = T$, the velocity of P is 6 m s^{-1} in the positive x-direction.

Find the value of T.

(6)

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2. A particle P of mass 0.6 kg is released from rest and slides down a line of greatest slope of a rough plane. The plane is inclined at 30° to the horizontal. When P has moved 12 m, its speed is 4 m s^{-1}. Given that friction is the only non-gravitational resistive force acting on P, find

(*a*) the work done against friction as the speed of P increases from 0 m s^{-1} to 4 m s^{-1},

(4)

(*b*) the coefficient of friction between the particle and the plane.

(4)

3.

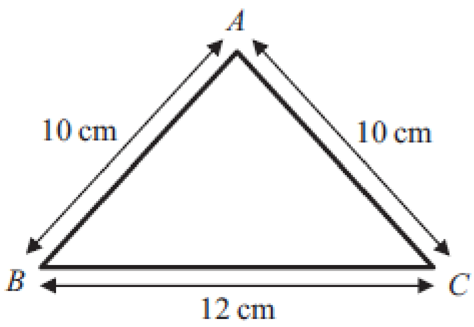

Figure 1

A triangular frame is formed by cutting a uniform rod into 3 pieces which are then joined to form a triangle ABC, where $AB = AC = 10$ cm and $BC = 12$ cm, as shown in Figure 1.

(*a*) Find the distance of the centre of mass of the frame from BC.

(5)

The frame has total mass M. A particle of mass M is attached to the frame at the mid-point of BC. The frame is then freely suspended from B and hangs in equilibrium.

(*b*) Find the size of the angle between BC and the vertical.

(4)

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4. A car of mass 750 kg is moving up a straight road inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{15}$. The resistance to motion of the car from non-gravitational forces has constant magnitude R newtons. The power developed by the car's engine is 15 kW and the car is moving at a constant speed of 20 m s^{-1}.

(*a*) Show that $R = 260$.

(4)

The power developed by the car's engine is now increased to 18 kW. The magnitude of the resistance to motion from non-gravitational forces remains at 260 N. At the instant when the car is moving up the road at 20 m s^{-1} the car's acceleration is a m s^{-2}.

(*b*) Find the value of a.

(4)

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5. [*In this question* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

A ball of mass 0.5 kg is moving with velocity (10**i** + 24**j**) m s^{-1} when it is struck by a bat. Immediately after the impact the ball is moving with velocity 20**i** m s^{-1}.

Find

(*a*) the magnitude of the impulse of the bat on the ball,

(4)

(*b*) the size of the angle between the vector **i** and the impulse exerted by the bat on the ball,

(2)

(*c*) the kinetic energy lost by the ball in the impact.

(3)

6.

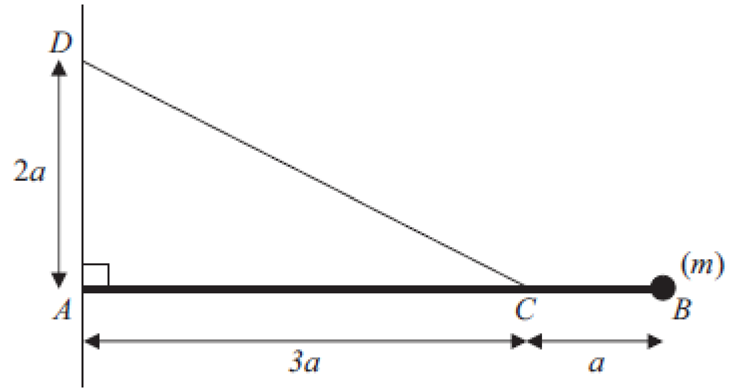


Figure 2

Figure 2 shows a uniform rod *AB* of mass *m* and length 4*a*. The end *A* of the rod is freely hinged to a point on a vertical wall. A particle of mass *m* is attached to the rod at *B*. One end of a light inextensible string is attached to the rod at *C*, where $AC = 3a$. The other end of the string is attached to the wall at *D*, where $AD = 2a$ and *D* is vertically above *A*. The rod rests horizontally in equilibrium in a vertical plane perpendicular to the wall and the tension in the string is *T*.

(*a*) Show that $T = mg\sqrt{13}$.

(5)

The particle of mass *m* at *B* is removed from the rod and replaced by a particle of mass *M* which is attached to the rod at *B*. The string breaks if the tension exceeds $2mg\sqrt{13}$. Given that the string does not break,

(*b*) show that $M \leq \frac{5}{2}m$.

(3)

7.

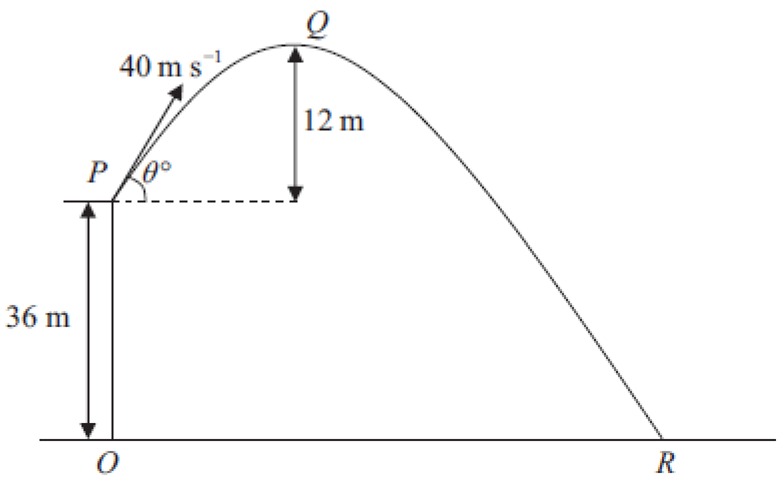


Figure 3

A ball is projected with speed 40 m s^{-1} from a point *P* on a cliff above horizontal ground. The point *O* on the ground is vertically below *P* and *OP* is 36 m. The ball is projected at an angle $\theta°$ to the horizontal. The point *Q* is the highest point of the path of the ball and is 12 m above the level of *P*. The ball moves freely under gravity and hits the ground at the point *R*, as shown in Figure 3. Find

(*a*) the value of θ,

(3)

(*b*) the distance *OR*,

(6)

(*c*) the speed of the ball as it hits the ground at *R*.

(3)

8. A small ball A of mass $3m$ is moving with speed u in a straight line on a smooth horizontal table. The ball collides directly with another small ball B of mass m moving with speed u towards A along the same straight line. The coefficient of restitution between A and B is $\frac{1}{2}$. The balls have the same radius and can be modelled as particles.

(*a*) Find

(i) the speed of A immediately after the collision,

(ii) the speed of B immediately after the collision.

(7)

After the collision B hits a smooth vertical wall which is perpendicular to the direction of motion of B. The coefficient of restitution between B and the wall is $\frac{2}{5}$.

(*b*) Find the speed of B immediately after hitting the wall.

(2)

The first collision between A and B occurred at a distance $4a$ from the wall. The balls collide again T seconds after the first collision.

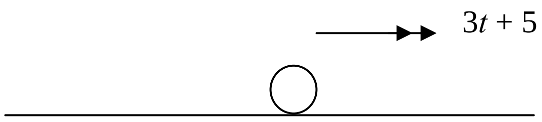
(*c*) Show that $T = \dfrac{112a}{15u}$.

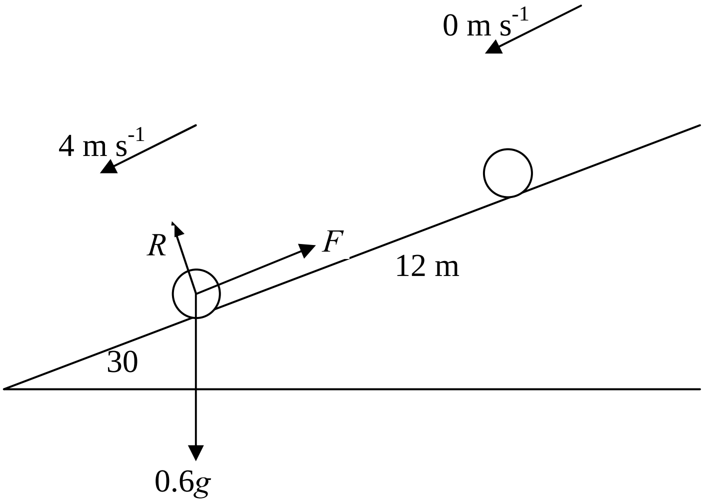
(6)

TOTAL FOR PAPER: 75 MARKS

END

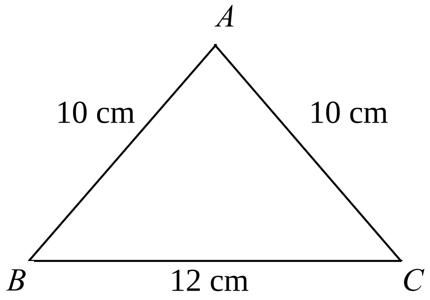
Summer 2010
Mechanics M2 6678
Mark Scheme

Question Number	Scheme	Marks
Q1		
	$\frac{\mathrm{d}v}{\mathrm{d}t} = 3t + 5$	
	$v = \int (3t+5)\,\mathrm{d}t$	M1*
	$v = \frac{3}{2}t^2 + 5t \quad (+c)$	A1
	$t = 0 \quad v = 2 \quad \Rightarrow \quad c = 2$	B1
	$v = \frac{3}{2}t^2 + 5t + 2$	
	$t = T \qquad 6 = \frac{3}{2}T^2 + 5T + 2$	DM1*
	$12 = 3T^2 + 10T + 4$	
	$3T^2 + 10T - 8 = 0$	
	$(3T-2)(T+4) = 0$	M1
	$T = \frac{2}{3} \quad (T = -4)$	
	$\therefore T = \frac{2}{3} \quad$ (or 0.67)	A1
		[6]

Question Number	Scheme	Marks
Q2		
(a)	K.E gained $= \frac{1}{2} \times 0.6 \times 4^2$ P.E. lost $= 0.6 \times g \times (12 \sin 30)$ Change in energy = P.E. lost - K.E. gained $= 0.6 \times g \times 12 \sin 30 - \frac{1}{2} \times 0.6 \times 4^2$ $= 30.48$ Work done against friction $= 30$ or 30.5 J	M1 A1 A1 A1 (4)
(b)	R$(\uparrow)$ $R = 0.6g \cos 30$ $F = \dfrac{30.48}{12}$ $F = \mu R$ $\mu = \dfrac{30.48}{12 \times 0.6g \cos 30}$ $\mu = 0.4987$ $\mu = 0.499$ or 0.50	B1 B1ft M1 A1 (4) **[8]**

Question Number	Scheme	Marks

Q3



(a)

	AB	*AC*	*BC*	frame		Marks
mass ratio	10	10	12	32		B1
dist. from *BC*	4	4	0	$\bar{x}$		B1

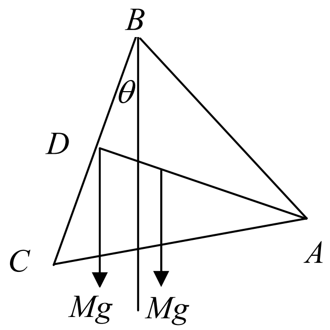
Moments about *BC*:

$$10\times4+10\times4+0=32\bar{x}$$ M1 A1

$$\bar{x}=\frac{80}{32}$$

$$\bar{x}=2\tfrac{1}{2}\quad(2.5)$$ A1 (5)

(b)



Moments about *B*:

$$Mg\times6\sin\theta=Mg\times\left(\bar{x}\cos\theta-6\sin\theta\right)$$ M1 A1 A1

$$12\sin\theta=\bar{x}\cos\theta$$

$$\tan\theta=\frac{\bar{x}}{12}$$

$$\theta=11.768.....=11.8^{\circ}$$ A1 (4)

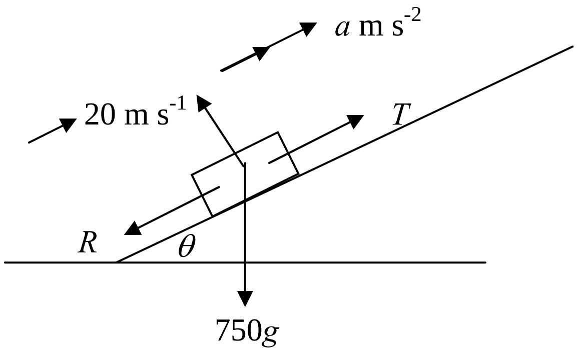
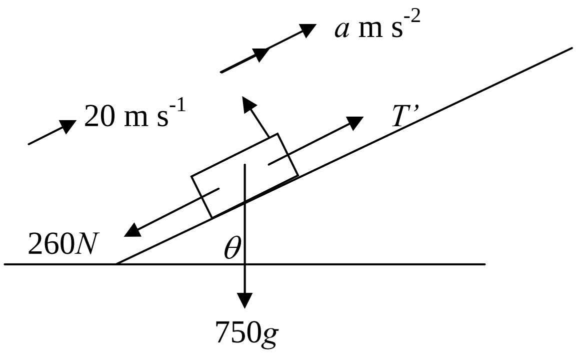
Alternative method :

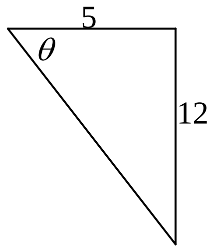
C of M of loaded frame at distance $\frac{1}{2}\bar{x}$ from *D* along *DA* B1

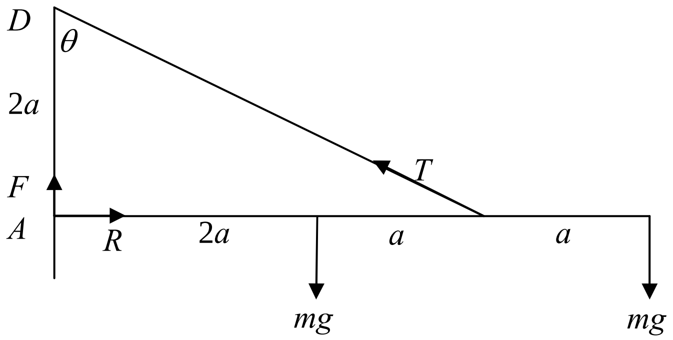
$$\tan\theta=\frac{\frac{1}{2}\bar{x}}{6}$$ M1 A1

$$\theta=11.768.....=11.8^{\circ}$$ A1

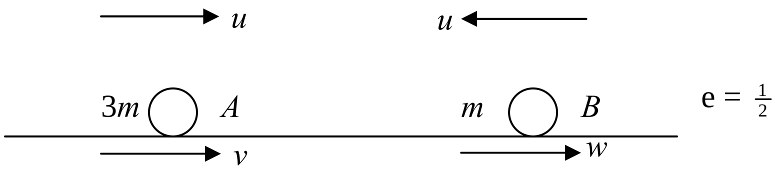
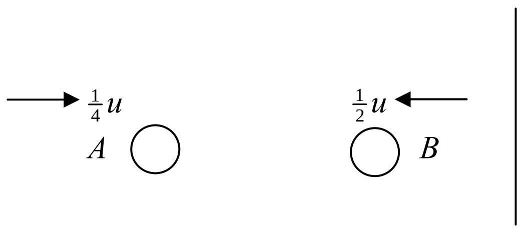
[9]

Question Number	Scheme	Marks
Q4		
(a)	$T = \dfrac{15000}{20} = 750$	M1
	R(parallel to road) $T = R + 750g\sin\theta$	M1 A1
	$R = 750 - 750 \times 9.8 \times \frac{1}{15}$	
	$R = 260*$	A1 (4)
(b)		
	$T' = \dfrac{18000}{20} = 900$	M1
	$T' - 260 - 750g \times \sin\theta = 750a$	M1 A1
	$a = \dfrac{900 - 260 - 750 \times 9.8 \times \frac{1}{15}}{750}$	
	$a = 0.2$	A1 (4)
		[8]

Question Number	Scheme	Marks
Q5 (a)	$\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ $= 0.5 \times 20\mathbf{i} - 0.5(10\mathbf{i} + 24\mathbf{j})$ $= 5\mathbf{i} - 12\mathbf{j}$ $\lvert 5\mathbf{i} - 12\mathbf{j} \rvert = 13$ Ns	 M1 A1 M1 A1 (4)
(b)	 $\tan\theta = \frac{12}{5}$ $\theta = 67.38$ $\theta = 67.4°$	 M1 A1 (2)
(c)	$\text{K.E.lost} = \frac{1}{2} \times 0.5(10^2 + 24^2) - \frac{1}{2} \times 0.5 \times 20^2$ $= 69$ J	M1 A1 A1 (3) **[9]**

Question Number	Scheme	Marks
Q6		
(a)	M(A) $3a \times T\cos\theta = 2amg + 4amg$ $\cos\theta = \left(\frac{2}{\sqrt{9+4}} =\right)\frac{2}{\sqrt{13}}$ $\frac{6}{\sqrt{13}}T = 6mg$ $T = mg\sqrt{13}$ *	M1 A1 A1 B1 A1 (5)
(b)	$3a \times T \times \cos\theta = 2amg + 4aMg$ $T = \frac{(2mg + 4Mg)}{6}\sqrt{13} \le 2mg\sqrt{13}$ $mg + 2Mg < 6mg$ $M \le \frac{5}{2}m$ * cso	M1 A1 A1 (3) **[8]**

Question Number	Scheme	Marks
Q7 (a)	Vertical motion: $v^2 = u^2 + 2as$ $(40\sin\theta)^2 = 2 \times g \times 12$ $(\sin\theta)^2 = \frac{2 \times g \times 12}{40^2}$ $\theta = 22.54 = 22.5°$ (accept 23)	M1 A1 A1 (3)
(b)	Vert motion $P \to R$: $s = ut + \frac{1}{2}at^2$ $-36 = 40\sin\theta t - \frac{g}{2}t^2$ $\frac{g}{2}t^2 - 40\sin\theta t - 36 = 0$ $t = \frac{40\sin 22.54 \pm \sqrt{(40\sin 22.54)^2 + 4 \times 4.9 \times 36}}{9.8}$ $t = 4.694...$ Horizontal P to R: $s = 40\cos\theta t$ $= 173$ m (or 170 m)	 M1 A1 A1 A1 M1 A1 (6)
(c)	Using Energy: $\frac{1}{2}mv^2 - \frac{1}{2}m \times 40^2 = m \times g \times 36$ $v^2 = 2\left(9.8 \times 36 + \frac{1}{2} \times 40^2\right)$ $v = 48.0\ldots..$ $v = 48$ m s^{-1} (accept 48.0)	M1 A1 A1 (3) **[12]**

Question Number	Scheme	Marks
Q8	 	
(a) (i)	Con. of Mom: $3mu - mu = 3mv + mw$	
	$2u = 3v + w$ (1)	M1# A1
	N.L.R: $\frac{1}{2}(u + u) = w - v$	M1# A1
	$u = w - v$ (2)	
	(1) – (2) $u = 4v$	DM1#
	$v = \frac{1}{4}u$	A1
(ii)	In (2) $u = w - \frac{1}{4}u$	
	$w = \frac{5}{4}u$	A1 (7)
(b)	B to wall: N.L.R: $\frac{5}{4}u \times \frac{2}{5} = V$	M1
	$V = \frac{1}{2}u$	A1ft (2)
(c)	 	
	B to wall: time $= 4a \div \frac{5}{4}u = \frac{16a}{5u}$	B1ft
	Dist. Travelled by $A = \frac{1}{4}u \times \frac{16a}{5u} = \frac{4}{5}a$	B1ft
	In t secs, A travels $\frac{1}{4}ut$, B travels $\frac{1}{2}ut$	
	Collide when speed of approach = $\frac{1}{2}ut + \frac{1}{4}ut$, distance to cover = $4a - \frac{4}{5}a$	M1$
	$\therefore\ t = \frac{4a - \frac{4}{5}a}{\frac{3}{4}u} = \frac{16a}{5} \times \frac{4}{3u} = \frac{64a}{15u}$	DM1$ A1
	Total time $= \frac{16a}{5u} + \frac{64a}{15u} = \frac{112a}{15u}$ *	A1 (6)
		15